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Reverse Bayesianism

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Definition

Reverse Bayesianism is a model of belief revision under growing awareness proposed by Karni and Vierø (2013). In the Bayesian paradigm, the decision-maker is aware of all possible states of the world. New information can only contract the state space by rendering null (ruling out) some events (sets of states) that were previously non-null, and beliefs update by redistributing the probability mass of the non-null events in the original state space proportionally among the remaining non-null events in the contracted state space. In the reverse Bayesian model, the decision-maker may be unaware of some of the possible states of the world. New discoveries can expand the state space by adding previously inconceivable events or by rendering non-null some events that were previously null (but conceivable), and beliefs update by redistributing probability mass proportionally away from the non-null events in the original state space to the new or newly non-null events in the expanded state space. This entry expounds the reverse Bayesian model of Karni and Vierø (2013) and points to related literature and legal applications.

Unawareness and Growing Awareness

Economists traditionally model choice under uncertainty according to Savage's (1954) theory of subjective expected utility. Savage's theory posits a space of mutually exclusive and collectively exhaustive states of the world, representing all possible resolutions of uncertainty. It assumes that when a person chooses an act, although she is uncertain about the true state of the world and therefore about the consequences of her chosen act, she nevertheless has complete knowledge of the state space – she is aware of all the possible acts and all the possible consequences of each and every act. New information can only contract the state space by rendering null (ruling out) some events (sets of states) that were previously non-null. In the wake of new information, a person's beliefs update according to Bayes' rule, which requires redistributing the probability mass of the non-null events in the original state space proportionally among the remaining non-null events in the contracted state space.

In reality, however, a person often does not have complete knowledge of the state space. This is known as unawareness. A person may be unaware of some acts, some consequences, or that a known act can cause a known consequence. Unawareness creates the possibility of growing awareness – the expansion of the state space when a person discovers a new act, consequence, or act-consequence link. Examples of growing awareness include the discovery of a new product

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or technology (new act), the discovery of a new disease or injury (new consequence), or the discovery of a new link between a known product and a known injury (new act-consequence link).

"Unawareness refers to the lack of conception rather than the lack of information" (Schipper 2014a, b). There is a fundamental difference between not knowing the state of the world (lack of information) and not knowing that a state of the world is possible (lack of conception). As noted above, the Savage model allows the state space to contract with the arrival of information and is consistent with Bayesian updating of beliefs. It, however, does not admit unawareness and cannot accommodate growing awareness (Dekel et al. 1998a, b).

The Reverse Bayesian Model

Karni and Vierø (2013) propose a model of belief revision under growing awareness called reverse Bayesianism. Reverse Bayesianism posits that when a person becomes aware of a new act, consequence, or act-consequence link, she revises her beliefs in a way that preserves the relative likelihoods of the events in the original state space. More specifically, the model postulates that (i) in the case of a new act or consequence, probability mass shifts proportionally away from the states in the original state space to the new states in the expanded state space, and (ii) in the case of a new act-consequence link, null states in the original state space become non-null, and probability mass shifts proportionally away from the original non-null states to the previously null states that become possible.

The primitives of the reverse Bayesian model are a finite, non-empty set F of feasible acts and a finite, non-empty set Z of feasible consequences. States are functions from the set of acts to the set of consequences. A state assigns a consequence to each act. The set of all possible states, Z^F, defines the conceivable state space. With m acts and n consequences, there are n^m conceivable states.

The decision-maker originally conceives the set of acts to be $F = \{f_1, \ldots, f_m\}$ and the set of consequences to be $Z = \{z_1, \ldots, z_n\}$. The conceivable state space is $Z^F = \{s_1, \ldots, s_{n^m}\}$, where each state $s = (s^1, \ldots, s^m) \in Z^F$ is a vector of length m, the ith component of which, s^i, is the consequence $z_j \in Z$ produced by act $f_j \in F$ in that state of the world.

An act-consequence link, or link, is a causal relationship between an act and a consequence. The conceivable state space admits all conceivable links. However, the decision-maker may perceive one or more links as infeasible, which brings her to nullify the states that admit such link. We refer to these as null states and denote them by $N \subset Z^F$. Taking only the non-null states defines the feasible state space $S \equiv Z^F \backslash N$. There are $\prod_{i=1}^{m}(n - \nu_i)$ feasible states, where ν_i denotes the number of nullified links involving act f_i.

The decision-maker's beliefs are represented by a probability measure p on the conceivable state space Z^F. The support of p is the feasible state space S. That is, $p(s) > 0$ for all $s \in S$ and $p(s) = 0$ for all $s \in N$.

The decision-maker may initially fail to conceive one or more acts or consequences or to perceive as feasible one or more conceivable links. We refer to such failures of conception or perception as unawareness. However, the decision-maker may later discover a new act or consequence, which expands both the feasible state space and the conceivable state space, or she may discover a new link, which expands the feasible state space but not the conceivable state space. (To be clear, by "new," we mean "not previously conceived" in the case of acts and consequences, and "previously conceived but perceived as infeasible" in the case of links.) We refer to such discoveries and expansions as growing awareness.

To illustrate, suppose $S = Z^F$ and the decision-maker discovers a new consequence z_{n+1}. Assuming the decision-maker links the new consequence to every act, the set of consequences becomes $\widehat{Z} = Z \cup \{z_{n+1}\}$ and the feasible and conceivable state spaces both expand to $\widehat{S} = \widehat{Z}^F = \{s_1, \ldots, s_{(n+1)^m}\}$, where each state remains a vector of length m. Alternatively, suppose the decision-maker discovers a new act, f_{m+1}. Then the set of acts becomes $\widehat{F} = F \cup \{f_{m+1}\}$ and, assuming the decision-maker links the new act to every consequence, the feasible and conceivable state spaces both expand to $\widehat{S} = Z^F = \{s_1, \ldots, s_{n^{(m+1)}}\}$, where each state now is a vector of length $m + 1$. Lastly, suppose $S \subset Z^F$ because (and only because) the decision-maker initially perceives as infeasible the link from f_1 to z_n. Discovery of the link from f_1 to z_n does not alter the conceivable state space, but the feasible state space expands to coincide with the conceivable state space: $\widehat{S} = Z^F$.

In the wake of growing awareness, the decision-maker revises her beliefs in a way that preserves the relative likelihoods of the events in the original feasible state space (the non-null events in the original conceivable state space). In each case of growing awareness, probability mass shifts proportionally away from the events in the original feasible state space to the new events in the expanded feasible state space. In the case of a new act or consequence, the new events in the expanded feasible state space are also new events in the expanded conceivable state space. In the case of a new link, the new events in the expanded feasible state space are the null events in the original conceivable state space that become non-null.

Karni and Vierø (2013) refer to this belief revision process as reverse Bayesianism. Let $\widehat{p}$ denote the decision-maker's revised beliefs on the expanded feasible state space $\widehat{S}$. Formally, reverse Bayesianism implies two restrictions on $\widehat{p}$: (i) in the case of a new consequence or link, $p(s)/p(t) = \widehat{p}(s)/\widehat{p}(t)$ for all $s,\ t \in S$; and (ii) in the case of a new act, $p(s)/p(t) = \widehat{p}(E(s))/\widehat{p}(E(t))$ for all $s,\ t \in S$, where $E(s)$ denotes the event in $\widehat{S}$ that corresponds to state $s \in S$; that is, given a new act $f_{m+1}, E(s) \equiv \left\{t \in \widehat{S} : t^i = s^i \text{ for all } i \neq m+1\right\}$.

The following section provides illustrations of conceivable and feasible state spaces, of state-space expansions due to the discovery of new acts, consequences, and links, and of reverse Bayesian updating in the wake of such discoveries.

Growing Awareness and Reverse Bayesian Updating

Consider a model with two acts, $F = \{f_1, f_2\}$, and two consequences, $Z = \{z_1, z_2\}$. The conceivable state space Z^F comprises four states: $s_1 = (z_1, z_1)$, $s_2 = (z_1, z_2)$, $s_3 = (z_2, z_1)$, and $s_4 = (z_2, z_2)$. The components of each state are the consequences produced by acts f_1 and f_2, respectively, in that state of the world. In state $s_3 = (z_2,\ z_1)$, for instance, act f_1 yields consequence z_2 and act f_2 yields consequence z_1. Let $p_k \equiv p(s_k), k = 1, \ldots, 4$, denote the decision-maker's beliefs on Z^F. We can depict the conceivable state space Z^F and the decision-maker's beliefs p as follows:

p	p_1	p_2	p_3	p_4
$F\backslash Z^F$	s_1	s_2	s_3	s_4
f_1	z_1	z_1	z_2	z_2
f_2	z_1	z_2	z_1	z_2

New Link

Suppose the decision-maker initially fails to conceive that act f_1 can yield consequence z_2. That is, suppose she initially perceives the event $\Delta = \{s_3,\ s_4\}$ as infeasible (null). This implies $p_3 = p_4 = 0$. We can depict the original feasible state space $S = \{s_1,\ s_1\} \subset Z^F$ and the decision-maker's initial beliefs p as follows:

p	p_1	p_2
$F\backslash S$	s_1	s_2
f_1	z_1	z_1
f_2	z_1	z_2

Suppose the decision-maker subsequently discovers that f_1 can yield z_2. The feasible state space expands to $\widehat{S} = S \cup \Delta$ and the decision-maker revises her beliefs from p to $\widehat{p}$:

$\widehat{p}$	$\widehat{p}_1$	$\widehat{p}_2$	$\widehat{p}_3$	$\widehat{p}_4$
$F\backslash \widehat{S}$	s_1	s_2	s_3	s_4
f_1	z_1	z_1	z_2	z_2
f_2	z_1	z_2	z_1	z_2

Reverse Bayesianism implies that the relative likelihood of states s_1 and s_2 remains unchanged: $p_1/p_2 = \widehat{p}_1/\widehat{p}_2$. Let δ denote the probability of the new event $\Delta = \{s_3, \ s_4\}$. Hence, $\delta = \widehat{p}_3 + \widehat{p}_4$. Note that $p_1 + p_2 = 1$ and $\widehat{p}_1 + \widehat{p}_2 = 1 - \delta$. It follows that $\widehat{p}_1 = (1-\delta)p_1$ and $\widehat{p}_2 = (1-\delta)p_2$. Reverse Bayesianism alone, however, does not pin down $\widehat{p}_3, \ \widehat{p}_4$, or δ.

Note that p is the Bayesian update of $\widehat{p}$ conditional on the event $S = \{s_1, s_2\}$ (i.e., the original feasible state space) – hence the term reverse Bayesianism.

New Act

Next, suppose the original feasible state space is the conceivable state space, i.e., $S = Z^F$. Suppose the decision-maker discovers a new act f_3 which she perceives can yield consequence z_1, z_2, or both. The expanded feasible state space is $\widehat{S} = \Delta_1 \cup \Delta_2$, where $\Delta_1 = \{s_1, \ s_2, \ s_3, \ s_4\}$ and $\Delta_2 = \{s_5, \ s_6, \ s_7, \ s_8\}$:

$\widehat{p}$	$\widehat{p}_1$	$\widehat{p}_2$	$\widehat{p}_3$	$\widehat{p}_4$	$\widehat{p}_5$	$\widehat{p}_6$	$\widehat{p}_7$	$\widehat{p}_8$
$F\backslash\widehat{S}$	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8
f_1	z_1	z_1	z_2	z_2	z_1	z_1	z_2	z_2
f_2	z_1	z_2	z_1	z_2	z_1	z_2	z_1	z_2
f_3	z_1	z_1	z_1	z_1	z_2	z_2	z_2	z_2

Observe that Δ_1 is an augmented copy of S in which f_3 yields z_1 in every state, and that Δ_2 is an augmented copy of S in which f_3 yields z_2 in every state. Stated differently, the expanded feasible state space $\widehat{S}$ is formed by splitting each state in S into two depending on whether f_3 yields z_1 or z_2. Hence, for each state s in S, there is a corresponding event $E(s)$ in $\widehat{S}$; specifically, $E(s_1) = \{s_1, s_5\}$, $E(s_2) = \{s_2, s_6\}$, $E(s_3) = \{s_3, s_7\}$, and $E(s_4) = \{s_4, s_8\}$. The connection between the sets of events $\{E(s_i): i = 1,\ldots,4\}$, and $\{\Delta_j : j = 1, \ 2\}$, both of which partition $\widehat{S}$, is that Δ_j collects the jth state from each $E(s_i)$.

Reverse Bayesiansim implies that the relative likelihoods of the states in S equal the relative likelihoods of their corresponding events in $\widehat{S}$. Thus,

$$\frac{p_1}{p_2} = \frac{\widehat{p}_1 + \widehat{p}_5}{\widehat{p}_2 + \widehat{p}_6}, \frac{p_1}{p_3} = \frac{\widehat{p}_1 + \widehat{p}_5}{\widehat{p}_3 + \widehat{p}_7}, \frac{p_1}{p_4} = \frac{\widehat{p}_1 + \widehat{p}_5}{\widehat{p}_4 + \widehat{p}_8},$$
$$\frac{p_2}{p_3} = \frac{\widehat{p}_2 + \widehat{p}_6}{\widehat{p}_3 + \widehat{p}_7}, \frac{p_2}{p_4} = \frac{\widehat{p}_2 + \widehat{p}_6}{\widehat{p}_4 + \widehat{p}_8}, \text{and } \frac{p_3}{p_4} = \frac{\widehat{p}_3 + \widehat{p}_7}{\widehat{p}_4 + \widehat{p}_8}.$$

It follows that reverse Bayesianism alone implies $\widehat{p}_1 + \widehat{p}_5 = p_1, \ \widehat{p}_2 + \widehat{p}_6 = p_2, \ \widehat{p}_3 + \widehat{p}_7 = p_3$, and $\widehat{p}_4 + \widehat{p}_8 = p_4$. However, it does pin down the individual probabilities of the states in $\widehat{S}$.

New Consequence

Last, suppose the original feasible state space is $S = Z^F$ and the decision-maker discovers a new consequence z_3 which she links to acts f_1 and f_2. The expanded feasible state space is $\widehat{S} = S \cup \Delta$, where $\Delta = \{s_5, \ s_6, \ s_7, \ s_8, \ s_9\}$:

$\widehat{p}$	$\widehat{p}_1$	$\widehat{p}_2$	$\widehat{p}_3$	$\widehat{p}_4$	$\widehat{p}_5$	$\widehat{p}_6$	$\widehat{p}_7$	$\widehat{p}_8$	$\widehat{p}_9$
$F\backslash\widehat{S}$	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9
f_1	z_1	z_1	z_2	z_2	z_3	z_3	z_1	z_2	z_3
f_2	z_1	z_2	z_1	z_2	z_1	z_2	z_3	z_3	z_3

Reverse Bayesianism implies that the relative likelihoods of the states in S remain unchanged. Hence, $p_1/p_2 = \widehat{p}_1/\widehat{p}_2, \ p_1/p_3 = \widehat{p}_1/\widehat{p}_3, \ p_1/p_4 = \widehat{p}_1/\widehat{p}_4, p_2/p_3 = \widehat{p}_2/\widehat{p}_3, \ p_2/p_4 = \widehat{p}_2/\widehat{p}_4$, and $p_3/p_4 = \widehat{p}_3/\widehat{p}_4$. Let δ denote the probability of the new event $\Delta = \{s_5, \ s_6, \ s_7, \ s_8, \ s_9\}$. Thus, $\delta = \widehat{p}_5 + \widehat{p}_6 + \widehat{p}_7 + \widehat{p}_8 + \widehat{p}_9$. Note that $p_1 + p_2 + p_3 + p_4 = 1$ and $\widehat{p}_1 + \widehat{p}_2 + \widehat{p}_3 + \widehat{p}_4 = 1 - \delta$. It follows that $\widehat{p}_1 = (1-\delta)p_1, \ \widehat{p}_2 = (1-\delta)p_2, \ \widehat{p}_3 = (1-\delta)p_3$, and $\widehat{p}_4 = (1-\delta)p_4$ Reverse Bayesianism alone, however, does not pin down $\widehat{p}_5, \ \widehat{p}_6, \ \widehat{p}_7, \ \widehat{p}_8, \ \widehat{p}_9$, or δ.

Related Literature and Legal Applications

The unawareness literature was pioneered by Fagin and Halpern (1988). Other early contributions include Modica and Rustichini (1994, 1999), Dekel et al. (1998b), Halpern (2001), Heifetz et al. (2006), and Halpern and Rêgo (2008). The early papers in the literature generally pursued an epistemic approach or a game-theoretic approach to modeling unawareness. Surveys of these papers are provided by Schipper (2014b) (which offers a “gentle introduction” to the literature) and Schipper (2015) (which provides an extended review).

Karni and Vierø (2013), who proposed the reverse Bayesian model, are among the pioneers of the choice-theoretic approach (i.e., the state-space approach) to modeling unawareness. Subsequent papers build on their approach. For instance, Grant et al. (2022) invoke their approach to model learning by experimentation in a world with unawareness; Karni and Vierø (2015, 2017) and Karni et al. (2021) extend the reverse Bayesian model to the cases where the decision-maker is probabilistically sophisticated (but does not necessarily abide by expected utility theory), where she anticipates her growing awareness, and where the discovery of new consequences nullifies some states that were non-null before the discovery; Dominiak and Tserenjigmid (2022) generalize the model such that the decision-maker perceives ambiguity in the wake of growing awareness; Chakravarty et al. (2022) provide conditions under which reverse Bayesianism fully determines the revised probability distribution over the expanded state space in each case of growing awareness; Becker et al. (2022) report experimental evidence that is consistent with reverse Bayesianism; and Schipper (2022) shows that prominent models of exchangeable random partitions, which are used to study discovery problems in other fields (e.g., the species discovery problem in biology), satisfy reverse Bayesianism. At the same time, Chambers and Hayashi (2018) criticize the empirical content of the reverse Bayesian model from a revealed preference perspective.

A handful of papers apply unawareness models to study legal topics. The bulk of these focus on contracts. For example, Zhao (2011) argues that unawareness may explain the existence of force majeure clauses in contracts; Filiz-Ozbay (2012) posits asymmetric awareness as a reason for the incompleteness of contracts; Grant et al. (2012) study aspects of differential awareness that give rise to contractual disputes; von Thadden and Zhao (2012, 2014) study the properties of optimal contracts under moral hazard when the agent may be partially unaware of her action space; Auster (2013) introduces asymmetric unawareness into the canonical moral hazard model and analyzes the properties of the optimal contract; and Board and Chung (2022) argue that asymmetric unawareness provides a justification for the contra proferentem doctrine of contract interpretation, which provides that ambiguous terms in a contract should be construed against the drafter. Focusing a different legal topic, Chakravarty et al. (2023) apply the reverse Bayesian model to study the implications of unawareness and growing awareness for tort law, and specifically for the negligence versus strict liability debate. They argue that negligence has an important advantage over strict liability in terms of spreading awareness about newly discovered accident risks. Future research could fruitfully apply the reverse Bayesian model to explore the ramifications of unawareness for additional legal topics such as contract remedies, criminal law, and litigation settlements.

Cross-References

- ▶ Choice Under Risk and Uncertainty
- ▶ Expected Utility Theory

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